Practical # 10

Statement:

Following the table give the consumption expenditures (C) and disposible income (Y) for sore families.

Regress C on Y and test for hetroscedasticity test (omit 4 values). Correct it for hetroscedasticity if it is found by assuming that error variance is proportion to Y^2.

C	Y	C	Y
106	120	150	180
108	122	159	190
114	130	169	200
117	134	172	204
123	140	175	207
130	150	179	209
133	155	182	210
138	160	185	213
142	166	188	216
144	170	191	220

Solution:

Goldfeld Quandt Test for Heteroscedasticity:

First of all,

Arrange the data w.r.t independent variable i-e "Y"

C	Y	
106	120	
108	122	$\hat{C}_1 = a_1 + b_1 Y_1$
114	130	
117	134	
123	140	
130	150	
133	155	
138	160	
142	160	
144	170	Omit 4 central values.
150	180	
159	190	
169	200	
172	204	
175	207	$\hat{C}_2 = a_2 + b_2 Y_2$
179	209	
182	210	
185	213	
188	216	
191	220	

Now, regress c_1 on Y_1 and c_2 on Y_2; we have,

c_1	Y_1	c_2	Y_2	$\hat{c}_1 = 11.86 + 0.78Y_1$	e_1^2	$\hat{c}_2 = -70.34 + 1.193Y_2$	e_2^2
106	120	169	200	106.3	0.09	168.26	0.5476
108	122	172	204	107.87	0.0159	173.032	1.0650
114	130	175	207	114.17	0.0289	176.611	2.5953
117	134	179	209	117.32	0.1011	178.997	0.000009
123	140	182	210	122.04	0.9216	180.19	3.2761
130	150	185	213	129.91	0.0081	183.769	1.5154
133	155	188	216	133.84	0.7140	187.348	0.4251
138	160	191	220	137.78	0.0484	192.12	1.2544
969	1111	1441	1679	—	1.928	—	10.68

From the data:

$\sum c_1 = 969$, $\sum c_2 = 1441$, $\sum Y_1 = 1111$, $\sum Y_2 = 1679$

$n_1 = 8$, $n_2 = 8$, $\sum c_1 Y_1 = 135809$, $\sum c_2 Y_2 = 302777$

$\sum c_1^2 = 118347$, $\sum c_2^2 = 259985$

$$\hat{c}_1 = a_1 + b_1 Y_1$$

$$b_1 = \frac{n_1 \sum c_1 Y_1 - (\sum c_1)(\sum Y_1)}{n_1 \sum Y_1^2 - (\sum Y_1)^2} = 0.787$$

$$a_1 = \bar{c}_1 - b_1 \bar{Y}_1 = 11.86$$

$$\hat{c}_1 = 11.86 + 0.787 Y_1$$

and;

$$\sum_{i=1}^{n_1} e_1^2 = \sum (c_1 - \hat{c}_1)^2$$

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$\hat{C}_2 = a_2 + b_2 Y_2$

$$b_2 = \frac{n_2 \sum Y_2 C_2 - (\sum Y_2)(\sum C_2)}{n_2 \sum Y_2^2 - (\sum Y_2)^2} = 1.193$$

$$a_2 = \bar{C}_2 - b_2 \bar{Y}_2 = -70.34$$

$$\hat{C}_2 = -70.34 + 1.193 Y_2$$

and,

$$\sum e_2^2 = \sum_{i=1}^{n_2} (C_2 - \hat{C}_2)^2$$

Testing Procedure:

Step 1:

Null and Alternative hypothesis are:

H_0: There is no heteroscedasticity

H_1: There is heteroscedasticity

Step 2:

Level of significance:

$\alpha = 0.05$

Step 3:

Test Statistic:

$$F = \frac{\sum e_2^2 / v_1}{\sum e_1^2 / v_2} = \frac{\sum e_2^2}{\sum e_1^2} \quad \because v_1 = v_2$$

Step 4:

Calculation:

$$F = \frac{10.68}{1.928} = 5.54$$

Step 5:

Critical Region:

Reject Ho if

$F \geq F_{\alpha, v_1, v_2}$

$F \geq F_{0.05(7,7)}$

$F \geq 3.787$

Step 6:

Conclusion:

As $5.54 > 3.787$, the calculated value of F falls in Rejection Region. So, reject Ho at 5% level of significance i-e there is hetroscedasticity in the data.

Remedial Measure for hetroscedasticity:

Given that

$Var(\epsilon_i) \propto Y^2$

$Var(\epsilon_i) = \sigma^2 Y^2$

$C_i = \alpha + \beta Y_i + \epsilon_i$

Divide by Y_i; we get

$$\frac{C_i}{Y_i} = \frac{\alpha}{Y_i} + \frac{\beta Y_i}{Y_i} + \frac{\epsilon_i}{Y_i}$$

$$w_i = \beta + \alpha V_i + U_i$$

where;

$$w_i = \frac{C_i}{Y_i} \quad \& \quad V_i = \frac{1}{Y_i}$$

$$\hat{\alpha} = \frac{n\sum w_i V_i - (\sum w_i)(\sum V_i)}{n\sum V_i^2 - (\sum V_i)^2} \quad \& \quad \beta = \bar{w} - \hat{\alpha}\bar{V}$$

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Now, the transformed variables are:

$w_1 = \frac{c_1}{y_1}$	$w_2 = \frac{c_2}{y_2}$	$v_1 = 1/y_1$	$v_2 = 1/y_2$	$\hat{w}_1$	$\hat{w}_2$	u_1^2	u_2^2
0.8833	0.8450	0.0083	0.005	0.8853	0.8441	0.000001	3.1×10^{-7}
0.8852	0.8431	0.0082	0.0049	0.8841	0.8500	1.21×10^{-6}	4.761×10^{-5}
0.8769	0.8454	0.0077	0.0048	0.8782	0.8560	1.69×10^{-6}	1.123×10^{-4}
0.8731	0.8565	0.0075	0.0048	0.8758	0.8560	7.29×10^{-6}	2.5×10^{-7}
0.8786	0.8667	0.0071	0.0048	0.8711	0.8560	5.62×10^{-5}	1.1449×10^{-4}
0.8667	0.8685	0.0067	0.0047	0.8663	0.8619	1.6×10^{-7}	4.356×10^{-5}
0.8581	0.8701	0.0065	0.0046	0.8639	0.8679	3.36×10^{-5}	7.84×10^{-6}
0.8625	0.8682	0.0063	0.0045	0.8616	0.8739	8.1×10^{-7}	3.24×10^{-5}
						1.05×10^{-4}	3.59×10^{-4}

$$\hat{w}_1 = 0.787 + 11.84 v_1$$

$$\hat{w}_2 = 1.143 - 59.79 v_2$$

Null & Alternative Hypothesis:

H_0: There is no hetroscedasticity.

H_1: There is hetroscedasticity.

level of significance:

$$\alpha = 0.05$$

Test Statistic:

$$F = \frac{\Sigma u_2^2}{\Sigma u_1^2}$$

Calculation: $F = \frac{3.59 \times 10^{-4}}{1.05 \times 10^{-4}} = 3.42$

Critical Region:

$$F \geq 3.787$$

Conclusion: Accept H_0 i.e there is no hetroscedasticity.

PRACTICAL # 11

Statement:

Assuming $Y_i = \alpha + \beta X_i + \varepsilon_i$

Apply Bartlet's Test for hetroscedasticity by dividing the data into three parts with No. of workers.

(i) 100, 150
(ii) 200, 250
(iii) 300, 350

X	Y					
100	84	85	86	87	89	90
150	89	91	93	93	94	96
200	95	98	99	103	104	105
250	103	106	109	113	115	117
300	116	118	121	125	127	131
350	136	128	130	132	134	137

Solution:

Testing Procedure:

(1) Null & Alternative hypothesis are:

H_0: There is no hetroscedasticity

H_1: There is hetroscedasticity

(2) Level of significance:

$$\alpha = 0.05$$

3) Test Statistic:

$$\chi^2 = \frac{v \ln \hat{\sigma}_p^2 - \ln v_j \hat{\sigma}_j^2}{1 + \frac{1}{3(m+1)}\left[\sum \frac{1}{v_j} - \frac{1}{v}\right]}$$

4) Calculation:

First of all, we have to compute the unbiased estimators of σ_1^2, σ_2^2 & σ_3^2.

we know that

$$\sigma_j^2 = \frac{SSE_j}{n-k-1}$$

$\because v_j = n_j - k - 1$

and also;

$$\hat{\sigma}_p^2 = \frac{\sum v_j \hat{\sigma}_j^2}{\sum v_j}$$

1st Group		2nd Group		3rd Group	
X_1	Y_1	X_2	Y_2	X_3	Y_3
100	84	200	95	300	116
100	85	200	98	300	118
100	86	200	99	300	121
100	87	200	103	300	125
100	89	200	104	300	127
100	90	200	105	300	131
150	89	250	103	350	126
150	91	250	106	350	128
150	93	250	109	350	130
150	93	250	113	350	132
150	94	250	115	350	134
150	96	250	117	350	137

$$Y_i = \alpha + \beta X_i + \varepsilon_i$$

$$\hat{\beta} = \frac{n\sum XY - (\sum X)(\sum Y)}{n\sum X^2 - (\sum X)^2}$$

$$\hat{\alpha} = \bar{Y} - \hat{\beta}\bar{X}$$

From the above data:

(i) $\sum X_1 = 1500$, $\sum Y_1 = 1077$, $\sum X_1^2 = 195000$,

$\sum Y_1^2 = 96819$, $\sum X_1 Y_1 = 135500$.

$$\hat{\beta}_1 = 0.117$$

$$\hat{\alpha}_1 = 75.17$$

$$\hat{Y}_1 = 75.17 + 0.117 X_1$$

$SSE_1 = \sum Y_1^2 - \hat{\alpha}_1 \sum Y_1 - \hat{\beta}_1 \sum X_1 Y_1$

$SSE_1 = 96819 - (75.17)(1077) - (0.117)(135500)$

$SSE_1 = 7.41$

$$\hat{\sigma}_1^2 = \frac{SSE_1}{n_1 - K - 1} = \frac{7.41}{12-1-1} = \frac{7.41}{10} = \boxed{0.741}$$

ii) $\sum X_2 = 3900$, $\sum Y_2 = 1267$, $\sum X_2^2 = 615000$, $\sum Y_2^2 = 134289$.

$\sum X_2 Y_2 = 286550$.

$\hat{\beta}_2 = 0.197$, $\hat{\alpha}_2 = 61.33$

$\hat{Y}_2 = 61.33 + 0.197 X_2$

$SSE_2 = \sum Y_2^2 - \hat{\alpha}_2 \sum Y_2 - \hat{\beta}_2 \sum X_2 Y_2$

$SSE_2 = 134289 - (61.33)(1267) - (0.197)(286550)$

$SSE_2 = 133.54$

$$\hat{\sigma}_2^2 = \frac{SSE_2}{n_2 - K - 1} = \frac{133.54}{12-1-1} = \boxed{13.354}$$

i) $\sum X_3 = 3900$, $\sum Y_3 = 1535$, $\sum X_3^2 = 1275000$

$\sum Y_3^2 = 196865$, $\sum X_3 Y_3 = 500350$

$\hat{\beta}_3 = 0.1633$, $\hat{\alpha}_3 = 74.00$

$\hat{Y}_3 = 74.0 + 0.1633 X_3$

$SSE_3 = \sum Y_3^2 - \hat{\alpha}_3 \sum Y_3 - \hat{\beta}_3 \sum X_3 Y_3$

$SSE_3 = 194245 - (74.0)(1525) - (0.1633)(496850)$

$SSE_3 = 259.395$

$$\hat{\sigma}_3^2 = \frac{SSE_3}{n_3 - K - 1} = \frac{259.395}{12-1-1} = \boxed{25.9395}$$

n_j	$v_j = d.f$	$\hat{\sigma}_j^2$	$v_j \hat{\sigma}_j^2$	$v_j \ln \hat{\sigma}_j^2$	$1/v_j$
12	10	0.741	7.41	-2.99	0.10
12	10	13.354	133.54	25.918	0.10
12	10	25.9395	259.395	32.56	0.10
			400.345	55.48	0.30

$$\hat{\sigma}_p^2 = \frac{\Sigma v_j \hat{\sigma}_j^2}{\Sigma v_j} = \frac{400.345}{30} = 13.345$$

$$\chi^2 = \frac{(30)\ln(13.345) - 55.48}{1 + \frac{1}{3(3-1)}\left(0.30 - \frac{1}{30}\right)}$$

$$\chi^2 = 5.89$$

(5) **Critical Region:**

Reject Ho if

$$\chi^2 \geq \chi^2_{\alpha, m-1}$$

$$\chi^2 \geq \chi^2_{0.05, 2}$$

$$\chi^2 \geq 5.99$$

(6) **Conclusion:**

As $5.89 < 5.99$, accept Ho at 5% level of significance i.e there is no heteroscedasticity in the data.

PRACTICAL # 12

Statement:

The following table gives the level of import (Y) and GDP (X), both seasonally adjusted in 100 billions from 1980 to 1991. Regress Y on X and test for auto-correlation by D.W test. Estimate first order autocorrelation by D.W statistic and correct for autocorrelation if any.

Y_t: 29.92 31.94 29.49 35.80 41.64 43.89 46.77
53.67 57.35 59.96 64.92 63.90 68.71 74.49
85.96 90.93 99.28 108.70 114.73 133.01.

X_t: 29.19 32.08 33.16 36.89 40.33 43.19 45.37
48.92 52.58 55.88 58.47 60.81 64.70 67.95
72.18 72.29 79.81 84.19 89.75 95.60.

Solution:

Y_t	X_t	Y_t	X_t
29.92	29.19	64.92	58.47
31.94	32.03	63.90	60.81
29.49	33.16	68.71	64.70
35.80	36.89	74.49	67.95
41.64	40.33	85.96	72.18
43.89	43.19	90.93	72.29
46.77	45.37	99.28	79.81
53.67	48.92	108.70	84.79
57.35	58.58	114.73	89.75
59.96	55.88	133.01	95.60

The Model is:

$$Y_t = \alpha + \beta X_t + \varepsilon_t$$

$\Sigma X_t = 1163.89$, $\Sigma X_t^2 = 75278.2425$, $\Sigma Y_t = 1335.06$

$\Sigma Y_t^2 = 106430.8862$, $\Sigma X_t Y_t = 89003.0925$

$$(\underset{\sim}{X}'\underset{\sim}{X}) = \begin{bmatrix} n & \Sigma X_t \\ \Sigma X_t & \Sigma X_t^2 \end{bmatrix} = \begin{bmatrix} 20 & 1163.89 \\ 1163.89 & 75278.2425 \end{bmatrix}$$

$$\underset{\sim}{X}'\underline{Y} = \begin{bmatrix} \Sigma Y_t \\ \Sigma X_t Y_t \end{bmatrix} = \begin{bmatrix} 1335.06 \\ 89003.0925 \end{bmatrix}$$

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$$(X'X)^{-1} = \begin{bmatrix} 0.4988 & -0.0077117 \\ -0.0077117 & 0.00013252 \end{bmatrix}$$

$$\beta = (X'X)^{-1} X'Y$$

$$\beta = \begin{bmatrix} 0.4988 & -0.0077117 \\ -0.0077117 & 0.00013252 \end{bmatrix} \begin{bmatrix} 1335.06 \\ 89003.0925 \end{bmatrix}$$

$$\beta = \begin{bmatrix} -20.4661 \\ 1.4988 \end{bmatrix}$$

Hence;

$$Y_t = -20.4661 + 1.4988 X_t$$

Testing Procedure:

) **Null & Alternative hypothesis are:**

H_0: There is no autocorrelation i.e $\rho = 0$.

H_1: There is autocorrelation i.e $\rho \neq 0$.

) **level of significance:**

$$\alpha = 0.05$$

Test Statistic:

$$d = \frac{\sum_{t=2}^{n} (e_t - e_{t-1})^2}{\sum_{t=1}^{n} e_t^2}$$

$$d = 2 - 2\hat{\rho} = 2(1-\hat{\rho})$$

$$\Rightarrow \hat{\rho} = \frac{2-d}{2}$$

$$\hat{\rho} = 1 - \frac{d}{2}.$$

(4) Calculation:

X_t	Y_t	$\hat{Y}_t = -20.46 + 1.49X_t$	e_t	e_{t-1}	$(e_t - e_{t-1})$
29.19	29.92	23.28	6.63	—	—
32.03	31.94	27.50	4.39	6.63	5.0024
33.16	29.49	29.23	0.25	4.39	17.1694
36.89	35.80	34.82	0.97	0.25	0.5177
40.33	41.64	39.98	1.65	0.97	0.4679
43.19	43.89	44.26	2.62	1.65	0.9281
45.37	46.77	47.53	-0.76	2.62	11.4745
48.92	53.67	52.85	0.81	-0.76	2.4942
52.58	57.35	58.34	-0.99	0.81	3.2602
55.88	59.96	63.28	-3.32	-0.99	5.4569
58.47	64.92	67.16	-2.24	-3.32	1.1623
60.81	63.90	70.67	-6.77	-2.24	20.4955
64.70	68.71	76.50	-7.79	-6.77	1.0412
76.95	74.49	81.37	-6.88	-7.79	0.8261
72.18	85.96	87.71	-1.75	-6.88	26.3179
72.29	90.93	87.88	3.04	-1.75	23.0889
79.81	99.28	99.15	0.12	3.04	8.5317
84.79	108.70	106.61	2.08	0.12	3.8255
89.75	114.73	114.05	0.67	2.08	1.9712
95.6	133.01	122.81	10.19	0.67	90.4781
			$\Sigma e^t = \ldots$		
			$\Sigma e^2 = 367.5741$		221.5097

$$\sum e_t^2 = 367.5741.$$

$$\Rightarrow \quad d = \frac{224.5097}{367.5741} = 0.6107$$

(5) Critical Region:

$$K = 1, \quad n = 20, \quad \alpha = 0.05$$

$$d_L = 1.201, \qquad d_U = 1.411.$$

Critical Region	No Conclusion	Acceptance Region	

d:	0	1.201 (d_L)	1.411 (d_U)	2	4
e:	1			0	-1

(6) Conclusion:

since $0.6107 < d_L$, so reject H_0 at 5% level of significance i-e there is positive autocorrelation in the data.

Now, for the correction:

As,

$$Y_t = \alpha + \beta X_t + \epsilon_t \quad \text{---(i)}$$

$$Y_{t-1} = \alpha + \beta X_{t-1} + \epsilon_{t-1} \quad \text{---(ii)}$$

$$\hat{e} Y_{t-1} = \hat{e}\alpha + \hat{e}\beta X_{t-1} + \hat{e}\epsilon_{t-1} \quad \text{---(iii)}$$

eq (i) − eq (iii)

$$Y_t - \hat{e} Y_{t-1} = \alpha(1 - \hat{e}) + \beta(X_t - \hat{e} X_{t-1}) + (\epsilon_t - \hat{e}\epsilon_{t-1})$$

$$Y_{*t} = \alpha_0 + \beta X_{*t} + u_t$$

as; $\hat{e} = 1 - \frac{d}{2} = 1 - \frac{0.6107}{2} = 0.695$

Y_t	Y_{t-1}	$Y_{*t} = Y_t - \hat{e}Y_{t-1}$	X_t	X_{t-1}	$X_{*t} = X_t - \hat{e}X_{t-1}$
29.92	—	—	29.19	—	—
31.94	29.92	11.1456	32.03	29.19	11.7429
29.49	31.94	7.2917	33.16	32.03	10.8992
35.80	29.49	15.3045	36.89	33.16	13.8438
41.64	35.80	16.7590	40.33	36.89	14.6915
43.89	41.64	14.9502	43.19	40.33	15.1607
46.77	43.89	16.2665	45.37	43.19	15.3529
53.67	46.77	21.1649	48.92	45.37	17.3879
57.35	53.67	20.0494	52.58	48.92	18.5806
59.96	57.35	20.1018	55.88	52.58	19.3369
64.92	59.96	23.2478	58.47	55.88	19.6334
63.90	64.92	18.7866	60.81	58.47	20.1734
68.71	63.90	24.2995	64.70	60.81	22.4371
74.49	68.71	26.7366	67.95	64.70	22.9835
85.96	74.49	34.1895	72.18	67.95	24.9548
90.93	85.96	31.1878	72.29	72.18	22.1249
99.28	90.93	36.0837	79.81	72.29	29.5685
108.70	99.28	39.7004	84.79	79.81	29.3221
114.73	108.70	39.1835	89.75	84.79	30.8209
133.01	114.73	53.2727	95.60	89.75	33.2238
		436.7157			392.2388

$\sum X_{*t}Y_{*t} = 11020.94984$, $\sum X_{*t}^2 = 8875.415$

$\bar{Y}_{*t} = 24.7218$, $\bar{X}_{*t} = 20.644$.

$$\hat{\beta} = \frac{\sum X_{*t} Y_{*t} - (\sum X_{*t})(\sum Y_{*t})/n}{\sum X_{*t}^2 - (\sum X_{*t})^2/n}$$

$$\hat{\beta} = \frac{11020.94984 - (392.23)(436.7157)/19}{8875.415 - (392.23)^2/19}$$

$$\hat{\beta} = 1.702$$

and,

$$\hat{\alpha}_0 = \bar{Y}_{*t} - \hat{\beta}\bar{X}_{*t}$$

$$\hat{\alpha}_0 = 24.7218 - (1.702)(20.644)$$

$$\hat{\alpha}_0 = -10.41305$$

Now;

$$\alpha = \frac{\hat{\alpha}_0}{1-\hat{e}} = \frac{-10.41305}{1-0.695}$$

$$\Rightarrow \alpha = -34.141$$

Hence, the estimated transformed model is:

$$Y_{*t} = -34.141 + 1.702 X_{*t}$$

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PRACTICAL # 13

Statement: A company which manufactures automative parts, wishes to model the effect of advertising on sales. The advertising expenditure each month (x) and the sales volume (Y) each month for the last two years are given as below.

Month	Y_t	X_t	Month	Y_t	X_t
1	92.8	25	13	85.4	15
2	79.2	0	14	80.4	5
3	84.5	15	15	83.5	10
4	83.0	10	16	92.5	25
5	88.1	20	17	89.5	15
6	83.9	10	18	83.6	5
7	79.7	5	19	89.1	15
8	81.1	5	20	90.9	20
9	86.4	15	21	92.7	25
10	86.3	15	22	88.1	15
11	79.9	5	23	79.5	0
12	86.6	20	24	82.9	5

Estimate the model; $Y_t = \beta_0 + \beta_1 X_t + \beta_2 t_2 + \epsilon_t$ where T= Month. Apply Geary test to check the presence of autocorrelation.

Estimate the autocorrelation co-efficsent by the Cochran orcutts two step procedure.

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Solution:

The estimated model is:

$$\hat{Y}_t = \hat{\beta}_0 + \beta_1 X_t + \hat{\beta}_2 t_2 + \varepsilon_t$$

From the data;

$\sum Y_t = 2049.8$, $\sum Y_t^2 = 175506.20$, $\sum X_t = 300.00$

$\sum X_t^2 = 5100$, $\sum X_t Y_t = 26344.5$, $\sum t_2 = 300.0$

$\sum t_2^2 = 4900$, $\sum X_t t_2 = 3745$, $\sum Y_t t_2 = 25755.90$

$$\hat{\beta} = (X'X)^{-1} X'Y$$

$$X'X = \begin{bmatrix} n & \sum X_t & \sum t_2 \\ \sum X_t & \sum X_t^2 & \sum X_t t_2 \\ \sum t_2 & \sum X_t t_2 & \sum t_2^2 \end{bmatrix} = \begin{bmatrix} 24 & 300 & 300 \\ 300 & 5100 & 3745 \\ 300 & 3745 & 4900 \end{bmatrix}$$

$$X'Y = \begin{bmatrix} \sum Y_t \\ \sum X_t Y_t \\ \sum t_2 Y_t \end{bmatrix} = \begin{bmatrix} 2049.8 \\ 26344.5 \\ 25755.9 \end{bmatrix} \quad \text{--- (1)}$$

$$(X'X)^{-1} = \begin{bmatrix} 0.29429 & -0.009299 & -0.01090 \\ -0.009299 & 0.00074075 & 3.22\times10^{-6} \\ -0.01090 & 3.22\times10^{-6} & 0.00086975 \end{bmatrix} \quad \text{--- (2)}$$

hence;

$$\hat{\beta} = \begin{bmatrix} 77.2386 \\ 0.5353 \\ 0.1183 \end{bmatrix} \quad \text{by (1) \& (2)}$$

The estimated model is:

$$Y_t = 77.2386 + 0.5353 X_t + 0.1183 t_2$$

Testing Procedure:

(1) **Null & Alternative hypothesis are:**

H_0: There is no autocorrelation i-e $\rho = 0$

H_1: There is autocorrelation i-e $\rho \neq 0$

(2) **Level of significance:**

$\alpha = 0.05$

(3) **Test Statistic:**

$$Z = \frac{R - Mean(R)}{S.E(R)}$$

$$Mean(R) = \frac{2N_1N_2}{N} + 1$$

$$S.E(R) = \frac{2N_1N_2(2N_1N_2 - N)}{N^2(N-1)}$$

(4) **Calculation:**

N_1 = Number of positive residuals.

N_2 = Number of negative residuals.

R = No. of Runs.

$N = N_1 + N_2$

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Y_t	X_t	t_2	$\hat{Y}_t = 77.23 + 0.535X_t + 0.118t_2$	$e_t = Y_t - \hat{Y}_t$
92.8	25	1	90.73	2.06
79.2	0	2	77.47	1.72
84.5	15	3	85.62	-1.12
83.0	10	4	83.04	-0.06
88.1	20	5	88.53	-0.43
83.9	10	6	83.30	0.59
79.9	5	7	80.74	-0.84
81.1	5	8	80.86	0.23
86.4	15	9	86.33	0.06
86.3	15	10	86.45	-0.15
79.9	5	11	81.21	-1.31
86.6	20	12	89.36	-2.76
85.4	15	13	86.80	-1.40
80.4	5	14	81.57	-1.17
83.5	10	15	84.36	-0.86
92.5	25	16	92.51	-0.013
89.5	15	17	87.27	2.22
83.6	5	18	82.04	1.55
89.1	15	19	87.51	1.58
90.9	20	20	90.31	0.58
92.7	25	21	93.10	-0.40
88.1	15	22	87.87	0.22
79.5	0	23	79.95	-0.45
82.9	5	24	82.75	0.14

here; $N_1 = 11$, $N_2 = 13$, $N = N_1 + N_2 = 24$.

$$Mean(R) = \frac{2(11)(13)}{24} + 1 = 12.9167.$$

$$Var(R) = \frac{2(11)(13)\{(2)(11)(13) - 24\}}{(24)^2(24-1)} = 5.6561.$$

$R = 11$, $S.E(R) = 2.378$

so;

$$Z = \frac{11 - 12.9169}{2.378} = -0.816.$$

(5) **Critical Region:**

Reject Ho if

$|Z| \geq Z_{\alpha/2}$

$|Z| \geq Z_{0.025}$

$|Z| \geq 1.96$

(6) **Conclusion:**

Since $Z_{cal} = -0.816$ falls in acceptance Region so accept Ho at 5% level of significance i-e there is no autocorrelation in the data.

Now, to estimate the correlation co-efficient by the Cochran Orcutts Two step procedure

$$\hat{e} = \frac{\sum e_t e_{t-1}}{\sum e_t^2}$$

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e_t	e_{t-1}	$e_t e_{t-1}$	e_t	e_{t-1}	$e_t e_{t-1}$
2.06	—	—	-1.40	-2.76	3.88
1.72	2.06	3.55	-1.17	-1.40	1.64
-1.12	1.72	-1.93	-0.86	-1.17	1.01
-0.06	-1.12	0.07	-0.013	-0.86	0.01
-0.43	-0.06	0.02	2.22	-0.013	-0.03
0.59	-0.43	-0.25	1.55	2.22	3.45
-0.84	0.59	-0.50	1.58	1.55	2.46
0.23	-0.84	-0.20	0.58	1.58	0.93
0.06	0.23	0.01	-0.40	0.58	-0.23
-0.15	0.06	-0.01	0.22	-0.40	-0.09
-1.31	-0.15	0.19	-0.45	0.22	-0.10
-2.76	-1.31	3.63	0.14	-0.45	-0.06

$$\sum e_t e_{t-1} = 17.4754$$

$$\sum e_t^2 = 33.9686$$

so;

$$\hat{e} = \frac{17.4754}{33.9689}$$

$$\hat{e} = 0.514$$

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